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THREE DIMENSIONAL ELASTO-VISCOPLASTIC FINITE ELEMENT ANALYSIS OF REINFORCED ROCK MASSES AND ITS APPLICATION

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SUMMARY

Passive, fully grouted rock bolts are widely used in rock engineering. Based on the observations of the laboratory test phenomena, a three-dimensional elasto-viscoplastic constitutive model which takes special attention to the bolt's behaviour at a joint has been proposed. The model is justified by the laboratory test and is applied to the structure design of the upper gate bay of the navigation lock in The Three Gorges Project. Copyright © 1999 John Wiley & Sons, Ltd.

Key words: finite element method; bolt; joint; rock; rheology

1. INTRODUCTION

The numerical evaluation of the safety of rock mass structure reinforced with passive, fully grouted bolts is one of the key problems in the massive rock excavation engineering. In the recent 30 years, many research results have been achieved which can be divided into two catalogues: one is the distinct modelling of bolts and joints[1–5], another one is the equivalent modelling,[6–11] either of them has special advantages, the former has the potentiality to describe the bolt's behaviour in very detail, and the latter can be applied to very complicated engineering problems with large number of joints and bolts.

The difficulties lie in the research of the numerical analysis for reinforced jointed rock masses are the simulation of the interfaces of bolt–grout and grout–rock as well as the simulation of the behaviour of bolt near joint. In 1989, Aydan[2] achieved a break-through on the distinct modelling by developing a three-dimensional bolt element with 8 nodal points. Two of these are connected to the bolt, whereas the six others are connected to rock mass. The number of nodes in the two-dimensional case is reduced to six. Swoboda and Marence,[3,4] modified the Aydan's formulation in two-dimensional case, assigning different co-ordinates for the bolt nodes and the nodes of

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rock–grout interfaces. Thus, the bolt's and the rock's displacements are different at the bolt–joint intersection. In this case, the parameters in the stiffness matrix are not constant. They depend on the joint displacement and are independently calculated by an iterative procedure. Chen and Egger[5] also developed a two-dimensional distinct model of bolt element with 6 nodes, of which two are connected to the bolt, whereas the four others are connected to rock mass, and a simplified analytical solution of the bolt's deformation at the bolt–joint intersection is used.

In the area of the equivalent modelling of bolts and joints, some useful and simple equivalent models also have been proposed. Zienkiewicz and Pande firstly proposed an elasto-viscoplastic 'multi-laminate' model for jointed rock masses. Later, Pande and Gerrard,[6] Pande and Sharma[7] modified the model to cover the reinforced cases. Larsson *et al.*[8,9] proposed a similar approach. In 1994, Chen and Pande[10] proposed a new rheological model which enables us to describe the bolt's behaviour in more detail even with an equivalent continuum approach. According to the observations of the laboratory tests,[12–15] the bolts have undergone an intensive local shear and tension deformation during the shearing of the joint. In the two-dimensional case, Egger and Chen[11] proved that such phenomenon can be included in the new rheological model to some extent if the test results can be correctly interpreted into the constitutive law of the bolted joint.

In this paper the authors try to establish a three-dimensional elasto-viscoplastic constitutive equation of jointed rock mass reinforced with passive, fully grouted bolts. The deduction is based on the new rheological model and special care has been taken for the bolt's behaviour at joint. The constitutive model has been implemented in a three-dimensional FEM program, the contrast between the results of laboratory test and numerical calculation is made to check the validity of the theory and the program. As the main practice motivation of the research, the complicated structure of the upper gate bay of the ship lock in The Three Gorges Project is studied.

2. RHEOLOGY MODEL AND PRINCIPLES

The rheological model shown in Figure 1 is firstly proposed by Chen and Pande,[10] the aim of the new model is to describe the bolt's behaviour in more detail with the equivalent continuum approach.

A global co-ordinate system is defined to formulate the overall constitutive equation, with its X-axis pointing northward, the Y-axis pointing westward, and the Z-axis being vertical. For each joint set and bolt set a local system is also needed to simplify the deduction. The local system of the joint set j is defined as follows (Figure 2) : the z_j-axis is perpendicular to the joint, the y_j-axis points in the direction of dip and x_j-axis is formed by the right-hand rule. The local system of the bolt set b is defined similarly (Figure 3): the z_b-axis is coincident with the bolt, the y_b-axis points in the direction of dip and the x_b-axis is formed by the right-hand rule.

All quantities without subscript mean the quantities of the equivalent material. Let the subscripts r, j, b denote the quantities of the rock material, the jth joint set and bth bolt set, respectively, the subscripts (br) and (bj) denote the quantities of the bth bolt in the rock material and the jth joint set; the subscripts $r(b)$ and $j(b)$ denote the quantities of the reinforced rock material and the reinforced jth joint set. All the above subscripts written in small letters imply that the quantities are in the local co-ordinate system, while the same subscripts written in capital letters imply that the quantities are in the global system.

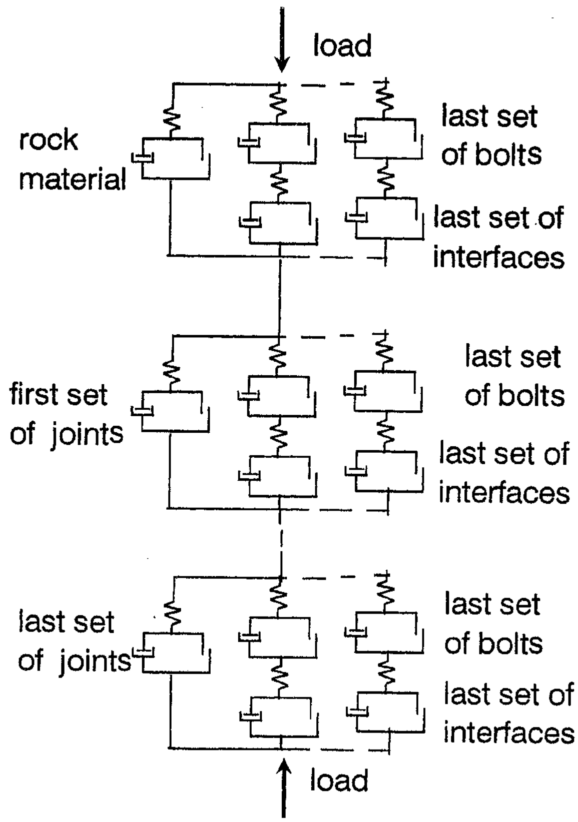

Figure 1. The new rheological model

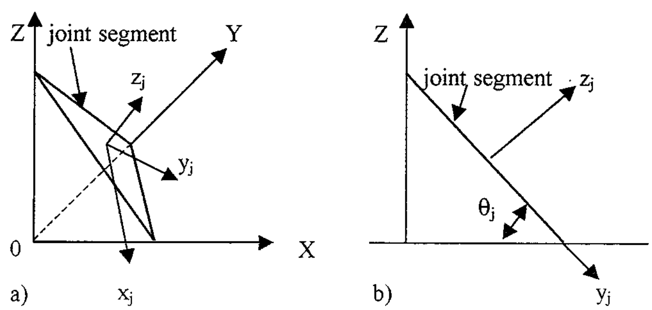

Figure 2. The local co-ordinate system of the joint set j: (a) the overall view; (b) the selection along the y_j–z_j plane

The strain and stress transforming between the global and the local coordinate systems is defined as follows:

$$\begin{aligned}\{\Delta\varepsilon\}_j &= [T]_j\{\Delta\varepsilon\}_J \\ \{\Delta\sigma\}_J &= [T]_j^{\mathrm{T}}\{\Delta\sigma\}_j\end{aligned} \tag{1}$$

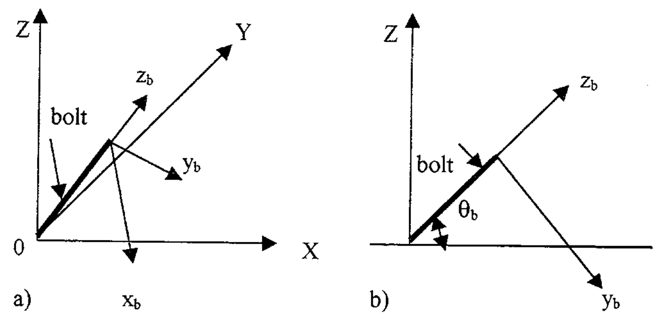

Figure 3. The local co-ordinate system of the bolt set b: (a) the overall view; (b) the selection along the y_b–z_b plane

$$\begin{aligned} \{\Delta\varepsilon\}_b &= [T]_b \{\Delta\varepsilon\}_B \\ \{\Delta\sigma\}_B &= [T]_b^{\mathrm{T}} \{\Delta\sigma\}_b \end{aligned} \tag{2}$$

The transforming matrices in equations (1) and (2) are

$$[T] = \begin{bmatrix} l_{11}^2 & l_{21}^2 & l_{31}^2 & l_{21}l_{31} & l_{11}l_{31} & l_{11}l_{21} \\ l_{12}^2 & l_{22}^2 & l_{32}^2 & l_{22}l_{32} & l_{12}l_{32} & l_{12}l_{22} \\ l_{13}^2 & l_{23}^2 & l_{33}^2 & l_{23}l_{33} & l_{13}l_{33} & l_{13}l_{23} \\ 2l_{12}l_{13} & 2l_{22}l_{23} & 2l_{32}l_{33} & l_{22}l_{33}+l_{23}l_{32} & l_{12}l_{33}+l_{32}l_{13} & l_{12}l_{23}+l_{22}l_{13} \\ 2l_{11}l_{13} & 2l_{21}l_{23} & 2l_{31}l_{33} & l_{21}l_{33}+l_{23}l_{31} & l_{11}l_{33}+l_{13}l_{31} & l_{11}l_{23}+l_{21}l_{13} \\ 2l_{11}l_{12} & 2l_{21}l_{22} & 2l_{31}l_{32} & l_{21}l_{32}+l_{22}l_{31} & l_{31}l_{12}+l_{11}l_{32} & l_{11}l_{22}+l_{21}l_{12} \end{bmatrix} \tag{3}$$

where

for the joint set j:

$$[L] = [L]_j = \begin{bmatrix} -\sin\phi_j & \cos\phi_j\cos\theta_j & \cos\phi_j\sin\theta_j \\ -\cos\phi_j & -\sin\phi_j\cos\theta_j & -\sin\phi_j\sin\theta_j \\ 0 & -\sin\theta_j & \cos\theta_j \end{bmatrix}$$

for the bolt set b: (4)

$$[L] = [L]_b = \begin{bmatrix} -\sin\phi_b & \cos\phi_b\sin\theta_b & \cos\phi_b\cos\theta_b \\ -\cos\phi_b & -\sin\phi_b\sin\theta_b & -\sin\phi_b\cos\theta_b \\ 0 & -\cos\theta_b & \sin\theta_b \end{bmatrix}$$

where ϕ_j, θ_j, ϕ_b, θ_b are the dip directions and dip angles of joint and bolt, respectively (see Figures 2 and 3).

The strain and stress transforming between the local co-ordinate systems of the joint set j and the bolt set b is defined as follows:

$$\begin{aligned} \{\Delta\varepsilon\}_b &= [T]_{(bj)} \{\Delta\varepsilon\}_{(bj)} \\ \{\Delta\sigma\}_{(bj)} &= [T]_{(bj)}^{\mathrm{T}} \{\Delta\sigma\}_b \end{aligned} \tag{5}$$

where:

$$[T]_{(bj)} = \begin{bmatrix} 0 \\ 0 \\ [T]^*_{(bj)} \\ 0 \end{bmatrix}$$

$$[T]^*_{(bj)} = \begin{bmatrix} l_{33}^2 & l_{33}l_{23} & l_{33}l_{13} \\ l_{33}l_{32} & l_{33}l_{22} & l_{33}l_{12} \\ l_{33}l_{31} & l_{33}l_{21} & l_{33}l_{11} \end{bmatrix} \tag{6}$$

$$[L] = [L]_j^{\mathrm{T}}[L]_b$$

For the present study the influence of the interfaces is not taken into consideration, so from the proposed rheological model four basic principles can be obtained for the jointed rock masses reinforced by the passive, fully grouted bolts:

(1) The strain increment of the bolted jointed rock is given as the sum of the incremental strains of the reinforced rock material and the reinforced joints.
(2) The load increment is shared among the bolt and the rock material in the reinforced rock material, the same applies to the reinforced joint.
(3) The mean stress in the bolted rock material is equal to the mean stress in the bolted joint.
(4) The strain of the bolt is equal to the strain of the rock material in the bolted rock material; similarly, the relative displacement of the two hinges is equal to the relative displacement of the joint.

Under the co-ordinate systems specified above, these principles can be formulated as follows:

(1)

$$\{\Delta\varepsilon\}^n = \{\Delta\varepsilon\}^n_{R(B)} + \sum_J \{\Delta\varepsilon\}^n_{J(B)} \tag{7}$$

(2)

$$\begin{aligned} \{\Delta\sigma\}^n_{R(B)} &= A_R\{\Delta\sigma\}^n_R + \sum_b A_b[T]_b^{\mathrm{T}}\{\Delta\sigma\}^n_{(br)} \\ \{\Delta\sigma\}^n_{j(b)} &= A_R\{\Delta\sigma\}^n_j + \sum_b A_b[T]^{\mathrm{T}}_{(bj)}\{\Delta\sigma\}^n_{(bj)} \end{aligned} \tag{8}$$

where A_R and A_b are the volumetric proportions of rock and bolt, with

$$A_R + \sum_b A_b = 1$$

(3)

$$\{\Delta\sigma\}^n = \{\Delta\sigma\}^n_{R(B)} = \{\Delta\sigma\}^n_{J(B)} \tag{9}$$

(4)

$$\begin{aligned} \{\Delta\varepsilon\}^n_{R(B)} &= \{\Delta\varepsilon\}^n_R = \{\Delta\varepsilon\}^n_{(BR)} \\ \{\Delta\varepsilon\}^n_{j(b)} &= \{\Delta\varepsilon\}^n_j = [T]^{-1}_{(bj)}\{\Delta\varepsilon\}^n_{(bj)} \end{aligned} \tag{10}$$

in which the reverse calculation $[T]^{-1}_{(bj)}$ can be carried out only for its submatrix $[T]^*_{(bj)}$ (equation (6)).

3. THE FORMULATION OF THE ELASTO-VISCOPLATIC CONSTITUTIVE EQUATION FOR BOLTED JOINTED ROCK MASSES

According to the elasto-viscoplastic potential theory,[16] at time t_n, the constitutive equation will take the following forms:

$$\begin{aligned}&\{\Delta\sigma\}^n = [D](\{\Delta\varepsilon\}^n - \{\dot{\varepsilon}^{vp}\}^n \Delta t_n)\\ &\text{or}\\ &\{\Delta\varepsilon\}^n = [S]\{\Delta\sigma\}^n + \{\dot{\varepsilon}^{vp}\}^n \Delta t_n\end{aligned} \tag{11}$$

where Δt_n is the time-stepping length, and $[D]$ and $[S]$ are the elastic matrix and compliance matrix, respectively. The viscoplastic flow rate is:

$$\{\dot{\varepsilon}^{vp}\}^n = \gamma \prec F \succ \left\{\frac{\partial Q}{\partial\{\sigma\}}\right\} \tag{12}$$

where γ is the fluidity parameter, F and Q are the yield and potential functions respectively, and the function $\langle F \rangle$ is defined as

$$\langle F \rangle = \begin{cases} F & \text{if} \quad F > 0 \\ 0 & \text{if} \quad F < 0 \end{cases}. \tag{13}$$

In the study of practice problems, if the fluidity parameter could be obtained by the laboratory and field tests or by the back analysis, the histories as well as the steady-state results of the deformation and failure of a structure could be calculated. However, in some cases it is not easy to get the appropriate fluidity parameter or only the elasto-plastic solution is of importance, under such circumstances we can simply assume that the fluidity parameter $\gamma = 1$. In this way the histories are not applicable, but the steady-state results of the deformation and failure are identical to the corresponding conventional static elasto-plastic solution.[16]

3.1. *The constitutive equation of intact rock*

The rock material is taken as an isotropic material whose elastic matrix is:

$$[D]_R = \begin{bmatrix} \lambda + 2G & \lambda & \lambda & 0 & 0 & 0 \\ & \lambda + 2G & \lambda & 0 & 0 & 0 \\ & & \lambda + 2G & 0 & 0 & 0 \\ & & & G & 0 & 0 \\ & \text{SYM} & & & G & 0 \\ & & & & & G \end{bmatrix} \tag{14}$$

For the yield of the intact rock, both the Mohr–Coulomb and the Drucker–Prager criteria are widely used; in the present study the latter is implemented in the FEM program:

$$\begin{aligned}&F_R = aI_1 + \sqrt{J_2} - k = 0\\ &a = \sin\varphi_R / \sqrt{3(3 + \sin^2\varphi_R)}\\ &k = \sqrt{3}c_R \cos\varphi_R / \sqrt{3 + \sin^2\varphi_R}\end{aligned} \tag{15}$$

The associate flow rule is adopted:

$$Q_R = F_R \tag{16}$$

so the flow rate of viscoplastic strain is

$$\{\dot{\varepsilon}^{\mathrm{vp}}\}_R^n = \gamma_R \prec F_R \succ \left\{\frac{\partial F_R}{\partial\{\sigma\}}\right\} \tag{17}$$

In equations (15)–(17) c_R, φ_R, γ_R are the cohesion, friction angle and fluidity parameter of intact rock, respectively.

3.2. The constitutive equation of joint

The relative displacement of the joint walls will be interpreted as equivalent strain according to the joint spacing. The elastic matrix of the jth joint in its local co-ordinate system can be expressed by the normal stiffness k_{nj}, the tangential stiffness k_{sj}, and the joint spacing d_j as

$$[D]_j = d_j \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{nj} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{sj} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{sj} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

Non-associate flow rule is considered for the joint:

$$\{\dot{\varepsilon}^{\mathrm{vp}}\}_j^n = \frac{\gamma_j}{d_j} \prec F_j \succ \left\{\frac{\partial Q_j}{\partial\{\sigma\}_j}\right\} \tag{19}$$

where the yield function F_j and potential function Q_j are expressed by the cohesion c_j, friction angle φ_j, dilatance angle ϕ_j and tension strength σ_{Tj}:

$$\begin{aligned} F_j &= \sqrt{\tau_{zxj}^2 + \tau_{zyj}^2} + \sigma_j \operatorname{tg}\varphi_j - c_j \quad \text{if } \sigma_j \prec \sigma_{Tj} \\ Q_j &= \sqrt{\tau_{zxj}^2 + \tau_{zyj}^2} + \sigma_j \operatorname{tg}\phi_j + \text{const.} \end{aligned} \tag{20}$$

$$\begin{aligned} F_j &= \sigma_j - \sigma_{Tj} \quad \text{if } \sigma_j \geqslant \sigma_{Tj} \\ Q_j &= \sqrt{\tau_{zxj}^2 + \tau_{zyj}^2 + \sigma_j^2} \end{aligned} + \text{const.} \tag{21}$$

3.3. The constitutive equation of Bolt

3.3.1. The bolt in a joint

A series of laboratory tests for a reinforced joint has been conducted at the Swiss Federal Institute of Technology in Lausanne[12–15]. The main conclusions can be extracted as follows by analysing the test results:

1. For samples with a bolt forming a small angle to the normal of the joint, bending of the bolt becomes predominant even when the shear force is small, which will create two hinges above

and below the joint plane (Figure 4). As the stress level is approximately constant in the zone between the two hinges, the exact location of failure cannot be predicted. Failure may occur by bending in one of the plastic hinges or by combined shear and tension near the shear surface. Test results showed the occurrence of both failure types for tests under apparently the same conditions, the ultimate loads and displacements at failure were nearly identical.

2. For samples with a bolt being inclined at a large angle to the normal of the joint, two hinges also developed in the tests, but the bending phenomena are not so strong. The great majority of the inclined bolts failed in tension near the shear surface.
3. The vertical height of the bended bolt is about 2–4 times the bolt diameter d_b, i.e. $h_b = 2 - 4d_b$. It is named 'effective height', corresponding to an 'effective length' of $L_b = h_b/\cos a_b$ (Figure 4). This height depends on the quality of the rock (or the grout mortar) and the bolt, on the bolt's diameter and the inclined angle, etc.
4. The sample in which the bolt forms a small angle with the normal of the joint show larger shear displacements, i.e. the inclined bolts react in a stiffer way than the normal ones. Besides, the maximum shear resistance of the bolted joint increases with the inclination of the bolt, provided there is considerable friction along the joint.
5. Large bolt diameters reduce the shear displacements required for obtaining a given shear force, and the maximum shear force increases linearly with the section of the bolt.

From the above conclusions it is natural to assume that there is an 'effective length' of the bolt at a joint (nearly the same quantity as the length between the hinges) within which nearly all deformations will take place (Figure 4). By the general assumption (1) of the equivalent material, the relative displacement between the two hinges is equal to the relative displacement of the joint walls at the intersection point. For the tension stress in this part, the uniform distribution along the effective length L_b seems a good and reasonable approach (Figure 5(a)), for the shear stress distribution we think two types are applicable as shown in Figures 5(b) and 5(c).

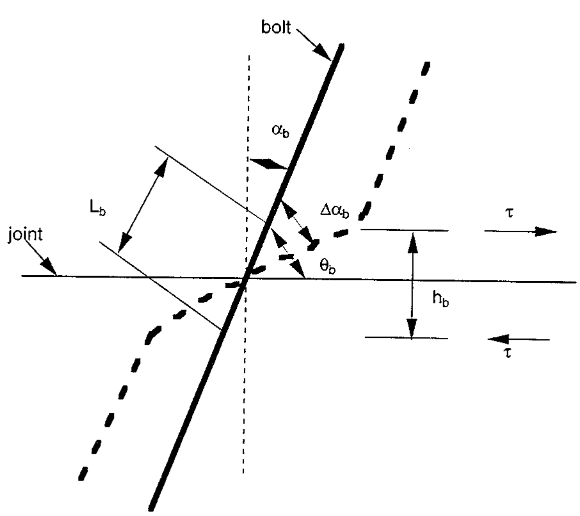

Figure 4. The deformation of the bolt near a joint

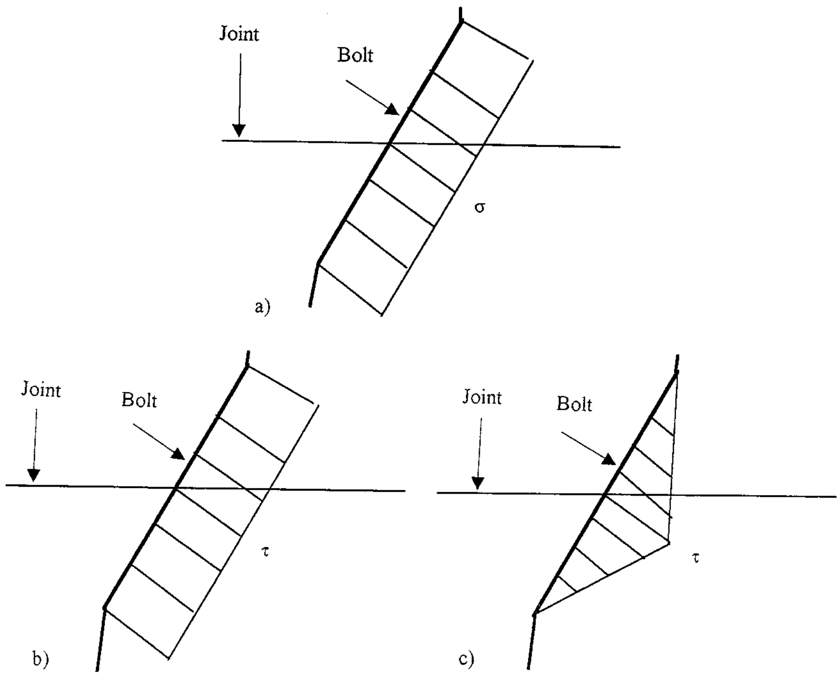

Figure 5. The ideal stresses distribution along the effective length of a bolt: (a) normal stress distribution; (b) shear stress distribution(uniform); (c) shear stress distribution(triangular)

The elastic relationship between strains and average stresses would be

$$\begin{aligned} \sigma_b &= E_b \varepsilon_b \\ \tau_b &= G_b \frac{1}{A\beta} \gamma_b \end{aligned} \tag{22}$$

where $A = 4/3$ for solid round bar, and:

$$\begin{aligned} &\beta = 1 \quad \text{for uniform shear stress distribution (Figure 5(b))} \\ &\beta = 1/2 \quad \text{for triangular shear stress distribution (Figure 5(c))} \end{aligned} \tag{23}$$

By what has been analysed above, it is clear that in the implementation of the numerical model more attention should be paid to the bolt's behaviour in the joint, especially the dip direction deflection $\Delta\phi_b$ and the dip angle defletion $\Delta\theta_b$ as well as the deformability of the bolt should be taken into account.

According to the above assumptions, on the bolt only normal stress σ_b and shear stress τ_{zxb}, τ_{zyb} can be transmitted (Figure 5), therefore the constitutive equation of the bolt set b at the joint set j expresssed in 'real' stress and strain would be

$$\{\Delta\sigma\}_b^{*_n} = [D]_{(bj)}^{*}(\{\Delta\varepsilon\}_b^{*_n} - \{\dot{\varepsilon}^{vp}\}_b^{*_n}\Delta t_n) \tag{24}$$

where

$$[D]^*_{(bj)} = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & [D]^{**}_{(bj)} & \\ & & & 0 \end{bmatrix}$$

$$[D]^{**}_{(bj)} = \begin{bmatrix} E_b & 0 & 0 \\ 0 & G_b/A/\beta & 0 \\ 0 & 0 & G_b/A/\beta \end{bmatrix} \tag{25}$$

$$\{\dot{\varepsilon}^{\mathrm{vp}}\}^{*_n}_b = \gamma_b \prec F_b \succ \left\{ \frac{\partial F_b}{\partial \{\sigma\}_b} \right\}$$

the Von Mises criterion with isotropic working hardening is used:

$$\begin{aligned} F_b &= [3(\tau^2_{zxb} + \tau^2_{zyb}) + \sigma^2_b]^{1/2} - \sigma \\ \sigma &= \sigma_y + (\sigma_u - \sigma_y)\gamma^{\mathrm{vp}}/{\gamma_u}^{\mathrm{vp}} \end{aligned} \tag{26}$$

in which σ_y, σ_u, γ^{vp}_u, γ^{vp} are the yield strength, ultimate strength, ultimate plastic general shear strain, and present plastic general shear strain respectively.

The strain increment of the bolt set b in the joint set j should be made equivalent to that of the rock mass according to the effective length and the joint spacing:

$$\begin{aligned} \{\Delta\varepsilon\}_b &= \frac{L_b}{d_j} \{\Delta\varepsilon\}^*_b \quad \text{or} \\ \{\Delta\varepsilon\}^*_b &= \frac{d_j}{L_b} \{\Delta\varepsilon\}_b \end{aligned} \tag{27}$$

Equation (24) becomes

$$\{\Delta\sigma\}^n_b = [D]_{(bj)}(\{\Delta\varepsilon\}^n_b - \{\dot{\varepsilon}^{\mathrm{vp}}\}^n_b \Delta t_n) \tag{28}$$

in which

$$\begin{aligned} [D]_{(bj)} &= \frac{d_j}{L_b} [D]^*_{(bj)} \\ \{\dot{\varepsilon}^{vp}\}^n_b &= \frac{L_b}{d_j} \{\dot{\varepsilon}^{vp}\}^{*_n}_b \end{aligned} \tag{29}$$

During the viscoplastic deformation, the length of hinge L_b, the dip direction ϕ_b and dip angle θ_b should be updated at definite time steps to simulate the effect of the change of the bolt's geometry characteristics. At any time step n, the corresponding quantities should be calculated as follows:

$$\begin{aligned} L^n_b &= L^{n-1}_b + d_j \Delta\varepsilon_b \\ \phi^n_b &= \phi^{n-1}_b + \operatorname{arctg}\left(\frac{d_j \Delta\gamma_{zxb}}{L^{n-1}_b} \right) \\ \theta^n_b &= \theta^{n-1}_b - \operatorname{arctg}\left(\frac{d_j \Delta\gamma_{zyb}}{L^{n-1}_b} \right) \end{aligned} \tag{30}$$

3.3.2. The bolt in the rock material. For the bolt set b in the rock material, it can be written directly:

$$\{\Delta\sigma\}^n_{(br)} = [D]_{(br)}(\{\Delta\varepsilon\}^n_{(br)} - \{\dot{\varepsilon}^{\mathrm{vp}}\}^n_{(br)}\Delta t_n) \tag{31}$$

where:

$$[D]_{(br)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & E_b & 0 & 0 & 0 \\ 0 & 0 & 0 & G_b & 0 & 0 \\ 0 & 0 & 0 & 0 & G_b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_{(br)} = \gamma_b \prec F_{(br)} \succ \left\{\frac{\partial F_{(br)}}{\partial\{\sigma\}_{(br)}}\right\} \tag{32}$$

3.4. The constitutive equation of equivalent material

Substituting the constitutive equation of each component into equation (8), and making strain transforming which is defined by equations (1)–(6), the constitutive equations of the bolted rock material and the bolted joint can be written as

$$\begin{aligned} \{\Delta\sigma\}^n_{R(B)} = {} & A_R[D]_R\{\Delta\varepsilon\}^n_R - A_R[D]_R\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_R\Delta t_n \\ & + \sum_b A_b[T]^{\mathrm{T}}_b[D]_{(br)}[T]_b\{\Delta\varepsilon\}^n_{(BR)} - \sum_b A_b[T]^{\mathrm{T}}_b[D]_{(br)}\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_{(br)}\Delta t_n \\ \{\Delta\sigma\}^n_{j(b)} = {} & A_R[D]_j\{\Delta\varepsilon\}^n_j - A_R[D]_j\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_j\Delta t_n \\ & + \sum_b A_b[T]^{\mathrm{T}}_{(bj)}[D]_{(bj)}[T]_{(bj)}\{\Delta\varepsilon\}^n_{(bj)} - \sum_b A_b[T]^{\mathrm{T}}_{(bj)}[D]_{(bj)}\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_b\Delta t_n \end{aligned} \tag{33}$$

Taking equation (10) into account, the above constitutive equation can be rewritten as

$$\begin{aligned} \{\Delta\sigma\}^n_{R(B)} &= [D]_{R(B)}\{\Delta\varepsilon\}^n_{R(B)} - \{\Delta\sigma^{\mathrm{vp}}\}^n_{R(B)} \\ \{\Delta\sigma\}^n_{j(b)} &= [D]_{j(b)}\{\Delta\varepsilon\}^n_{j(b)} - \{\Delta\sigma^{\mathrm{vp}}\}^n_{j(b)} \end{aligned} \tag{34}$$

or

$$\begin{aligned} \{\Delta\varepsilon\}^n_{R(B)} &= [D]^{-1}_{R(B)}\{\Delta\sigma\}^n_{R(B)} + [D]^{-1}_{R(B)}\{\Delta\sigma^{\mathrm{vp}}\}^n_{R(B)} \\ \{\Delta\varepsilon\}^n_{j(b)} &= [D]^{-1}_{j(b)}\{\Delta\sigma\}^n_{j(b)} + [D]^{-1}_{j(b)}\{\Delta\sigma^{\mathrm{vp}}\}^n_{j(b)} \end{aligned} \tag{35}$$

where

$$\begin{aligned} [D]_{R(B)} &= A_R[D]_R + \sum_b A_b[T]^{\mathrm{T}}_b[D]_{(br)}[T]_b \\ [D]_{j(b)} &= A_R[D]_j + \sum_b A_b[T]^{\mathrm{T}}_{(bj)}[D]_{(bj)}[T]_{(bj)} \end{aligned} \tag{36}$$

and

$$\begin{aligned}\{\Delta\sigma^{\mathrm{vp}}\}^n_{R(B)} &= A_R[D]_R\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_R\Delta t_n + \sum_b A_b[T]^{\mathrm{T}}_b[D]_{(br)}\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_{(br)}\Delta t_n \\ \{\Delta\sigma^{\mathrm{vp}}\}^n_{j(b)} &= A_R[D]_j\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_j\Delta t_n + \sum_b A_b[T]^{\mathrm{T}}_{(bj)}[D]_{(bj)}\{\dot{\varepsilon}^{\mathrm{vp}}\}^n_b\Delta t_n\end{aligned} \tag{37}$$

Transforming the constitutive equation of the bolted joint in equation (35) using the matrix $[T]_j$ defined in equation (1), then putting it into equation (7) together with the constitutive equation of the bolted rock material, the constitutive equation of the bolted jointed rock masses can be obtained:

$$\{\Delta\varepsilon\}^n = [S]\{\Delta\sigma\}^n + \{\Delta\varepsilon^{\mathrm{vp}}\}^n \tag{38}$$

where

$$[S] = [D]^{-1}_{R(B)} + \sum_b([T]_j)^{-1}[D]^{-1}_{j(b)}([T]^{\mathrm{T}}_j)^{-1} \tag{39}$$

and

$$\{\Delta\varepsilon^{\mathrm{vp}}\}^n = [D]^{-1}_{R(B)}\{\Delta\sigma^{\mathrm{vp}}\}^n_{R(B)} + \sum_j[T]^{-1}_j[D]^{-1}_{j(b)}\{\Delta\sigma^{\mathrm{vp}}\}^n_{j(b)} \tag{40}$$

The above constitutive model has been implemented in a three-dimensional FEM program CORE3 in which the common 8-node hexahedral iso-parametric element is used, the formulation of the FEM for the elasto-viscoplastic problem is the same as in the literature.[16]

4. EVALUATION OF THE THEORY BY LABORATORY TEST

4.1. Test sample and mesh

The direct shear sample analysed is made of concrete with dimensions of 220 mm × 200 mm × 150 mm.[12] The mesh used in the calculation is shown in Figure 6(a) (projected at the X–Z plane, thickness = 200 mm in direction of Y-axis). Since the 'equivalent continuum' approach is adopted in the program, and there are only one joint and one bolt in the sample, therefore it is essential to deploy thin elements along the joint and bolt (Figure 6(b)). For the element containing the joint, the thickness of the element should be specified as the joint spacing d; similarly, for the element containing the bolt the volumetric proportion should be specified as the ratio of bolt's volume to the element's volume.

In Table I the parameters of the concrete block are listed, the parameters of the joint and the bolt are listed in Table II and Table III. Since only the elasto-plastic behaviour of the sample is concerned in the test, the fluidity parameters of the concrete and the joint as well as the bolt are all assumed as 1·0 in the calculation which has been discussed before.

In the laboratory test and numerical analysis, a normal pressure $\sigma = 0{\cdot}2$ MPa is applied at the top of the sample first, then the shear force T is increased step by step until the sample fails. Different values of the 'effective length' or 'effective height' are tried to obtain the best fitting of the shear force vs displacement curve. The effective height varies from two times to five times the bolt diameter, i.e. $h_b = 2 \sim 4d_b$. The shear force T vs. displacement u_x relationships for different 'effective heights' are given in Figure 7.

It can be seen that, if the 'effective length' or 'effective height' bolt at joint could be correctly specified by the tests, both the deformation and strength of the reinforced joint can be well predicted.

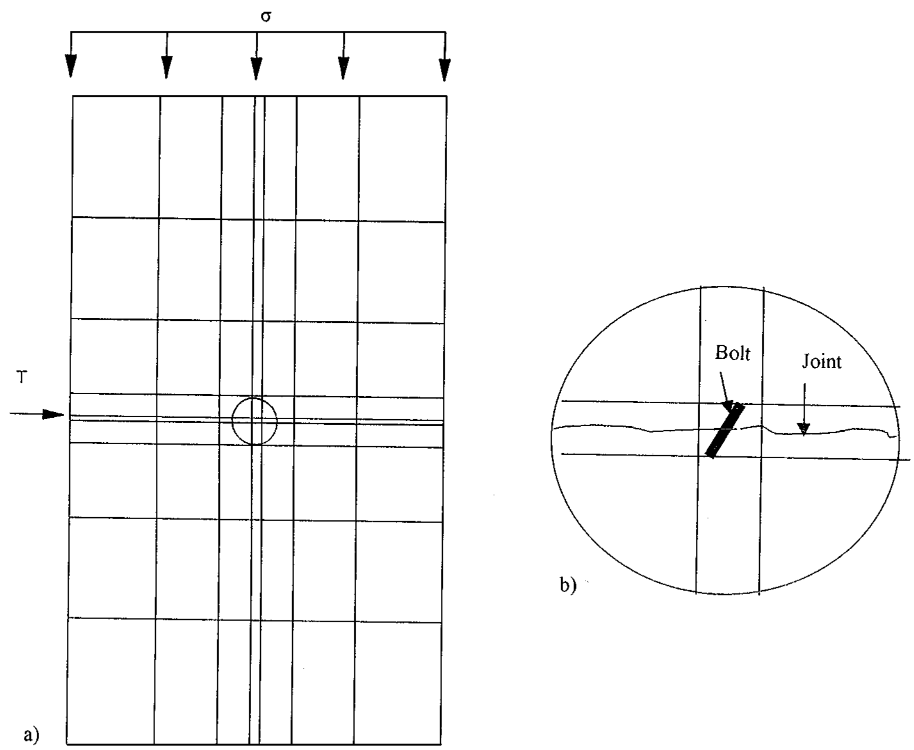

Figure 6. Mesh of the test sample: (a) the projection of the mesh at the X–Z plane; (b) the details of the element containing one joint and one bolt

Table I. Parameters of concrete

E (MPa)	μ	c(MPa)	φ (deg)	ϕ (deg)
17800·0	0·2	12·5	45·0	45·0

Table II. Parameters of joint

kn (MN/m^3)	ks (MN/m^3)	c (MPa)	φ (deg)	ϕ (deg)
1000	300	0	34	0

Table III. Parameters bolt

E (GPa)	μ	σ_y (MPa)	σ_u (MPa)	γ^{vp} (%)	ϕ (deg)	θ (deg)	d (mm)
210	0·3	630	670	27	0	60	8

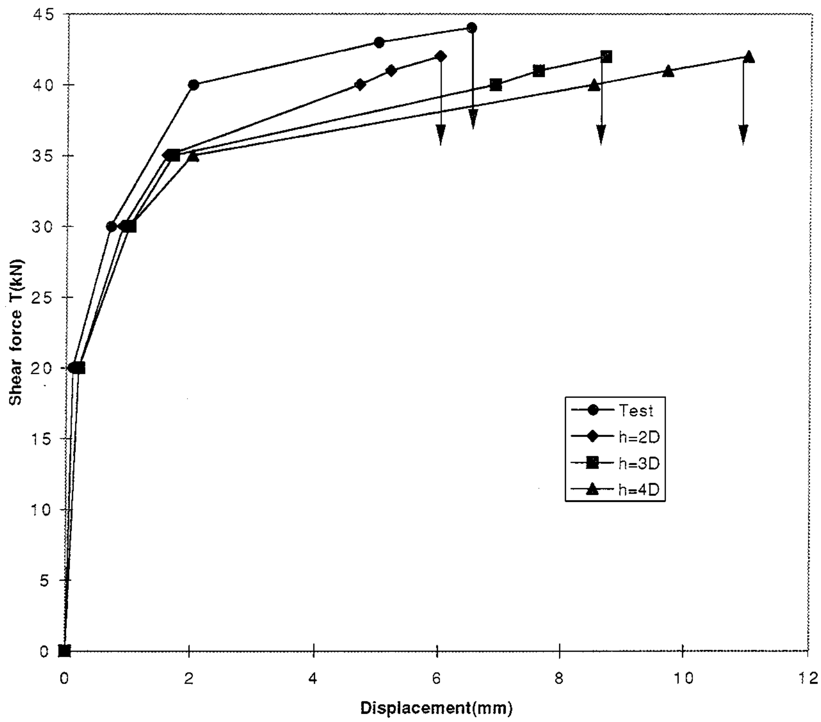

Figure 7. Shear force T vs. displacement u_x of bolted joint (dip angle $\theta_b = 60°$)

5. APPLICATION TO THE COMPLICATED ENGINEERING STRUCTURE

Figure 8 is the plan of the upper gate bay of the double track navigation lock in The Three Gorges Project. The gate bay is excavated in the granite rock with dimensions of 70 m (length) × 58 m (width) × 66 m (height). The concrete retaining walls and floor are reinforced together with rock mass by systematic bolts. The bolts have two roles to play: in the construction period, they share the excavation loads to improve the stability of the rock slope; in the operation period, they share the hydrostatic pressure, thrust from the gates, seepage pressure, etc., acting on the concrete walls to ensure the safety of the whole gate bay structure. To get the rational design of the bolting and structure, three-dimensional finite element analysis has been conducted in which the reinforced joints between concrete wall and rock is simulated with the method developed in this paper.

Figure 9 is the finite element mesh including 3212 elements and 4252 nodes. Similar to the study of the test sample (see Figure 6), thin elements along the contact joints between the concrete wall and the rock masses are deployed, while for the three main sets of joint and the bolts in the slope rocks the real equivalent approach can be realized, therefore, in the generation of the grid no special attention should be paid for their position and the orientation.

The excavating, bolting, and concrete refilling are all simulated by seven stages, for each stage the construction elevation is above 185, 185–170, 170–160, 160–150, 150–140, 140–130 m, below 130 m. The main loads considered are: the weight of the concrete; the hydrostatic pressure in the

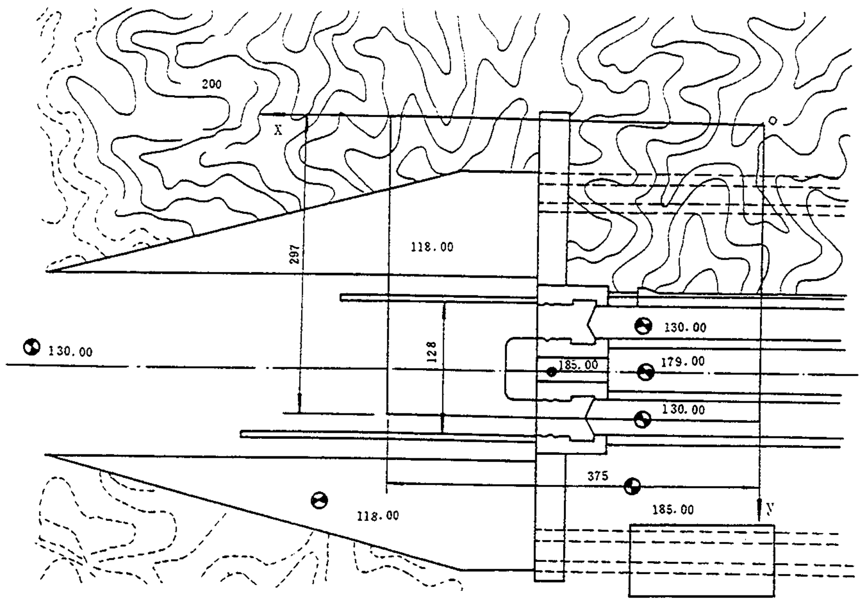

Figure 8. The plan of the gate bay of the double-track ship lock

gate bay; the seepage pressure behind the retaining wall and under the floor; the thrust forces from the miter gates.

Again, no appropriate fluidity parameters can be used in the present study, the program CORE3 is only used to get the elasto-plastic solution of the structure, so in the calculation the fluidity parameters of the rocks and the joints as well as the bolts are all taken as 1.

5.1. *Excavation period*

(a) After the completion of the excavation, the maximum displacement of rock slope takes place at the top of the wall (Figure 10). The maximum displacement is 3 cm if there are no systematic bolts. However, the maximum displacement will be reduced to 2 cm if the effect of the systematic bolt is taken into account. Therefore, it can be affirmed that the systematic bolting is very important to the stability of rock slope during excavation.
(b) After the completion of the excavtaion, the maximum principal stress appears at the left bottom corner of the gate bay (Figure 11), the maximum principal stress value is 24·56 Mpa which is much lower than the long-term creep strength of the rock (≈ 50 Mpa), hence there is no danger of hazardous creep deformation of rock slope which could influence the operation of the miter gates.
(c) After the completion of the excavation, the maximum tension and shear stresses in bolts are 54·2 and 40·1 Mpa, respectively.

5.2. *Concrete refilling period*

The refilling of the concrete has great influence on the bolt's stress at the contact joint between the concrete wall and the rock, but has very minor influence on the stresses in the rock slope. After

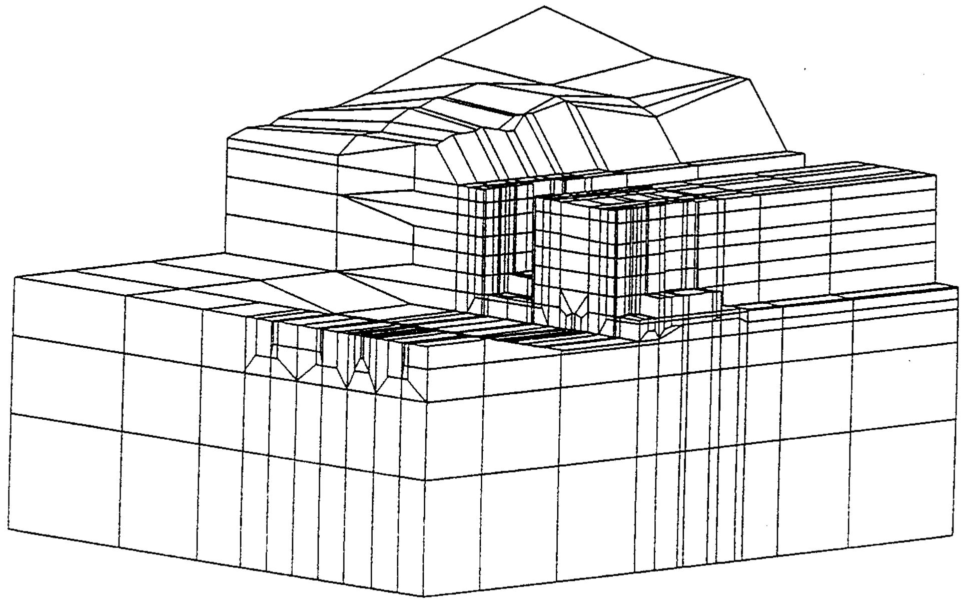

Figure 9. The finite element mesh for calculation

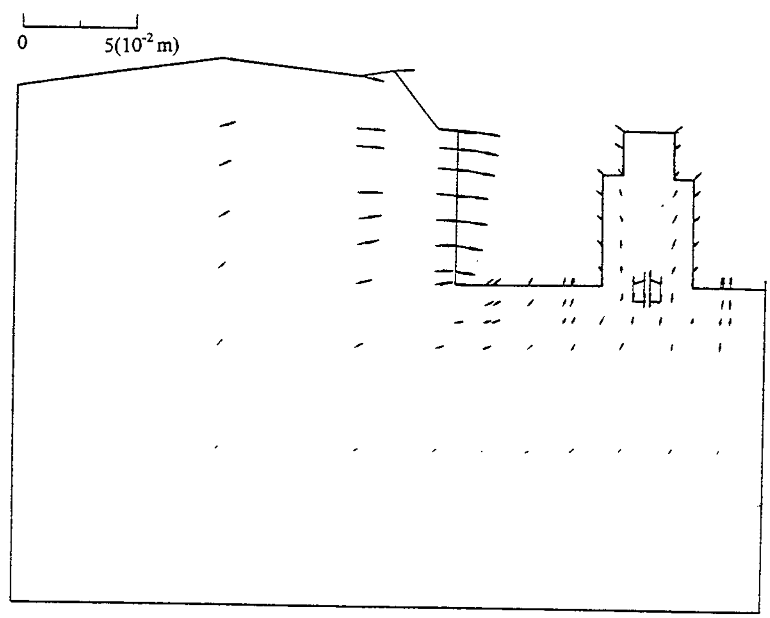

Figure 10. Displacement induced by the excavation

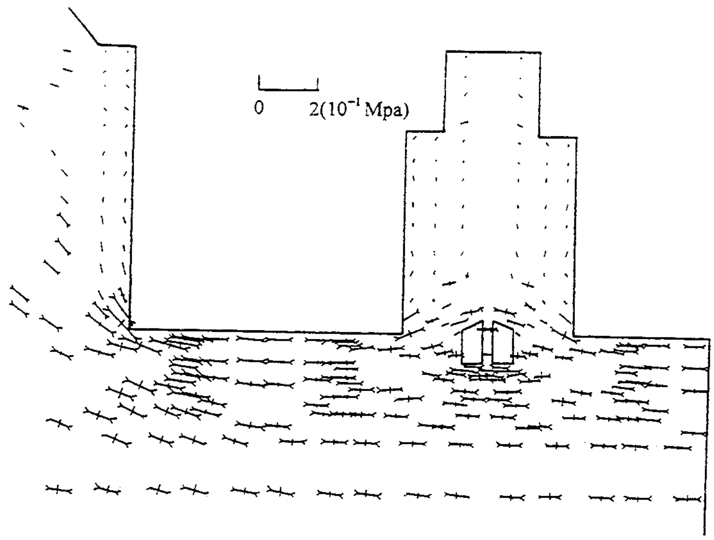

Figure 11. Principal stress after the completion of the excavation

the completion of the refilling, the maximum compression and shear stresses of the bolts at the joint are 178.3 and 65 Mpa, respectively.

5.3. Operation period

1. Under any circumstances, there is little change of stresses in slope rock, but there is a wide variation of bolt's stresses at the contact joint between concrete wall and slope rock, the main influence factors are the seepage pressure behind the concrete wall and the thrust forces from the miter gates. The maximum tension and shear stresses of bolts at the joint are 178·3 and 165 Mpa, respectively; therefore it can be confirmed that the minimum safety factor of bolts is greater than 3.
2. The maximum displacement of the concrete wall is 0·5 mm under the different load combinations, and under any circumstances the concrete wall works in the elastic state. By the contrast, the calculation result shows that the concrete retaining wall will fail if there are no systematic bolts to join the wall and the slope rock together.
3. Since the maximum bolt stresses always occur at the middle elevation or above the middle elevation of the concrete wall, so it is suggested that more bolts should be deployed at the elevation 135–150 m, the density of bolts below the elevation 135 m can be cut dawn.

6. CONCLUSIONS

Based on a new rheological model and laboratory tests, a three-dimensional elasto-viscoplastic constitutive equation for the reinforced rock masses is formulated. The main advantage of the proposed formulation is that the bolt's behaviour can be described in more detail even with the equivalent continuum approach. However, the adoption of the four basic principles in the deduction imposes a restriction on the proposed constitutive equation that it only can be used in the cases when the bolts are passive and fully grouted.

The constitutive equation has been implemented in a FEM program whose validity is verified by the contrast between the results of laboratory test and numerical analysis. The analysis of the gate bay of the double-track ship lock of The Three Gorges Project shows the ability of the proposed constitutive model and the FEM program in handling the complicated rock engineering problems.

The work presented in this paper is only a part of series works. To make a thorough study of the slope and gate bay referred to in this paper, the following are noteworthy in the future works:

(a) The rational and practical three-dimensional modelling of the reinforced jointed rock masses with the combination of distinct and equivalent approach.
(b) The correct simulation of systematic pre-stressed anchor cables, in this case the mechanical properties of the interfaces of anchor-grout and grout-rock can not be neglected.

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